Worksheet 2

Revision: Exponents and surds

No calculators!

1. Simplify the expressions.

1.1 $6^2 - 2^6 + 3^0$

1.2 $4^3 - 3^4 - \left(\frac{1}{2}\right)^{-1}$

1.3 $7^2 + 2^7 - (-1)^{14}$

1.4 $3^3 \times 3^{-1} \times 3^0$

1.5 $2^3 \times 4^3 \times 2^{-6}$

1.6 $9^2 \div 3^2 \div 3^{-1}$

1.7 $(5^2 \cdot 2^3 \cdot 3^{-2})^0$

1.8 $(2^{-1} + 3^{-1})^2$

2. Simplify the expressions with rational exponents.

2.1 $\left(8^{\frac{1}{3}}\right)\left(25^{\frac{3}{2}}\right)\left(16^{\frac{3}{4}}\right)$

2.2 $9^{\frac{3}{2}} + 8^{\frac{2}{3}}$

2.3 $64^{\frac{1}{2}} - 64^{\frac{1}{3}}$

2.4 $18^{\frac{1}{2}} \times 2^{\frac{1}{2}}$

2.5 $\left(125^{\frac{1}{3}} \times 5\right)^{\frac{1}{2}}$

2.6 $\left(27^{\frac{2}{3}} - 1\right)^{-\frac{1}{3}}$

2.7 $36^{\frac{1}{2}} - 16^{\frac{3}{4}} + 8^{\frac{2}{3}}$

2.8 $\left(\frac{1}{4}\right)^{-\frac{1}{2}} + \left(\frac{1}{9}\right)^{-\frac{1}{2}}$

3. Simplify the surds.

3.1 $\sqrt{2}\sqrt{8} + \sqrt[3]{64}$

3.2 $\sqrt{4 \times 9} - \sqrt[3]{5 \times 25}$

3.3 $\sqrt{9} + \sqrt{16} + \sqrt{9 + 16}$

3.4 $\sqrt{16 \times 9} - \sqrt{\sqrt[3]{64}}$

3.5 $\sqrt{\frac{25}{9}} + \sqrt{2\frac{1}{4}}$

3.6 $2\sqrt{3} \times 3\sqrt{12}$

3.7 $\left(\sqrt[3]{4}\sqrt[3]{16}\right)^2$

3.8 $\sqrt[3]{-27} + \sqrt[5]{32}$

3.9 $\sqrt[10]{1} - \sqrt[11]{-1}$

3.10 $\sqrt[3]{-8\,000} + \sqrt[2]{400}$

Using the laws to simplify powers

For example: Simplify: $\frac{8^n \cdot 6^{n-3} \cdot 9^{1-n}}{3^{-n} \cdot 16^{n-1}}$

Step 1: Write numerical bases as a product of their prime factors:

$$\frac{8^n \cdot 6^{n-3} \cdot 9^{1-n}}{3^{-n} \cdot 16^{n-1}}$$

$$= \frac{(2^3)^n (2)^{n-3} (3)^{n-3} (3^2)^{1-n}}{3^{-n} \cdot (2^4)^{n-1}}$$

Step 2: Use the exponent laws to simplify numerator and denominator:

$$= \frac{2^{3n} 2^{n-3} 3^{n-3} 3^{2-2n}}{3^{-n} \cdot 2^{4n-4}}$$

$$= \frac{2^{3n+n-3} \cdot 3^{n-3+2-2n}}{3^{-n} \cdot 2^{4n-4}}$$

$$= 2^{4n-3-4n+4} \cdot 3^{-1-n+n}$$

$$= 2^1 \cdot 3^{-1}$$

$$= \frac{2}{3}$$

1. Simplify the powers with rational exponents. Leave the answers with positive exponents.

1.1 $x^4 \cdot x^6 \cdot x^{-3}$ **1.2** $\frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{4}}}{x^{\frac{1}{6}}}$ **1.3** $\left(x^{\frac{2}{3}}\right)^6$ **1.4** $\frac{a^{-4}}{a^{-6}}$

1.5 $\frac{x^{-7}}{x^0}$ **1.6** $\frac{x^3y^2}{x^2y^3}$ **1.7** $\frac{a^{-1}b^{-2}c}{a^{-2}b^2c^{-1}}$ **1.8** $\left(a^{\frac{1}{2}}\right)^6$

1.9 $(x^{-2})^{-3} \div \left(\frac{1}{x^{-3}}\right)^{-1}$ **1.10** $\frac{(2a)^3}{(3a)^2}$

2. Simplify the powers with rational exponents. Leave the answers with positive exponents.

2.1 $\frac{25a^2b^3}{15ab^2}$ **2.2** $\frac{3^2a^{\frac{1}{3}}2a^{\frac{2}{3}}b^{\frac{1}{2}}}{6b^{-\frac{1}{2}}}$

2.3 $\frac{a^2b^3c}{abc} \times \frac{ab^2c^3}{b^3c^2}$ **2.4** $\left(\frac{25a^2b^2}{5ab^3}\right)^2$

2.5 $\left(\frac{64x^6}{y^3}\right)^{\frac{1}{3}}$ **2.6** $\frac{3(2x)^3}{2(3x)^2}$

2.7 $\left(\frac{15^x3^x}{9^{x+1}5^{x-2}}\right)^{\frac{1}{2}}$ **2.8** $\frac{5^{x-1}25^x}{5^{3x+2}}$

2.9 $\frac{4^{x-2}16^{x+1}}{8^{2x+1}}$ **2.10** $\frac{6^{3x}12^{-x}4^x}{24^{2x+1}8^{-x}}$

Using the laws to simplify surds

For example: Simplify: $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{32}}$

Step 1: Write each number as a product of a square number and another number:

$$\frac{\sqrt{50} + \sqrt{18}}{\sqrt{32}}$$

$$= \frac{\sqrt{2 \times 25} + \sqrt{2 \times 9}}{\sqrt{2 \times 16}}$$

Step 2: Use the surd laws to separate the square roots:

$$\frac{\sqrt{2 \times 25} + \sqrt{2 \times 9}}{\sqrt{2 \times 16}}$$

$$= \frac{\sqrt{2}\sqrt{25} + \sqrt{2}\sqrt{9}}{\sqrt{2}\sqrt{16}}$$

$$= \frac{5\sqrt{2} + 3\sqrt{2}}{4\sqrt{2}}$$

Step 3: Simplify:

$$\frac{5\sqrt{2} + 3\sqrt{2}}{4\sqrt{2}}$$

$$= \frac{8\sqrt{2}}{4\sqrt{2}} = 2$$

1. Simplify without using a calculator. Leave your answer in simplest surd form, where necessary.

1.1 $\sqrt{12} + \sqrt{48} - \sqrt{27}$ **1.2** $\sqrt{500} - \sqrt{125} - \sqrt{20}$

1.3 $\sqrt{8} + \sqrt{18} - \sqrt{72}$ **1.4** $\sqrt{2} - \sqrt{32} + \sqrt{162} - \sqrt{50}$

1.5 $\frac{\sqrt{32} + \sqrt{8}}{\sqrt{2}}$ **1.6** $\sqrt[5]{32} \times \frac{1}{\sqrt[3]{8}}$

1.7 $3\sqrt{2} + \sqrt{8} - \sqrt{50} + \frac{\sqrt{72}}{\sqrt{18}} - \sqrt{\sqrt[3]{64}}$ **1.8** $3\sqrt{12} \times 2\sqrt{3}$

1.9 $\frac{\sqrt{75} + \sqrt{27}}{\sqrt{48}}$ **1.10** $3\sqrt{45} - 2\sqrt{80}$

2. Simplify without using a calculator. Leave your answer in simplest surd form, where necessary.

2.1 $\sqrt{5}(2\sqrt{45} - \sqrt{125})$ **2.2** $(\sqrt{3} + 2)(\sqrt{3} - 2)$

2.3 $\left(\sqrt[3]{\sqrt{71} - \sqrt{7}}\right)\left(\sqrt[3]{\sqrt{71} + \sqrt{7}}\right)$ **2.4** $\frac{6 + \sqrt{18}}{3}$

2.5 $\sqrt{\frac{2^{43} + 2^{40}}{2^{40}}}$ **2.6** $\sqrt{\sqrt{36} - 2}$

2.7 $\sqrt{20x^6 - 4x^6}$ **2.8** $\sqrt{128x^6} + \sqrt{98x^6}$

2.9 $\sqrt{\frac{2x^4 + 3x^4}{x^2}}$ **2.10** $\sqrt{\frac{3^2a^8 + 2^4a^8}{a^4}}$

3. Between which two integers does each surd lie?

3.1 $\sqrt{134}$ **3.2** $-\sqrt{14}$ **3.3** $\sqrt[3]{43}$ **3.4** $\sqrt[4]{55}$

Solving exponential equations

Variable in the exponent

When the variable is part of the exponent, isolate the power and get the bases the same on both sides of the equation. Then drop the bases and equate the exponents to solve for x.

For example: Solve for x in the equation: $3 \cdot 2^x - 5 = 91$.

Step 1: Isolate the variable term:

$3 \cdot 2^x = 91 + 5$

$\therefore 3 \cdot 2^x = 96$

Step 2: Divide both sides of the equation by 3 to isolate the power:

$2^x = 96 \div 3$

$\therefore 2^x = 32$

Step 3: Write the terms on both sides of the equation with equal bases:

$2^x = 2^5$

Step 4: Drop the bases and equate the exponents: $x = 5$.

Solving for the exponent x

Solve the equations.

1. **1.1** $5^x = 625$ **1.2** $3 \cdot 5^x = 75$

1.3 $2 \cdot 3^x = 162$ **1.4** $4 \cdot 2^x = 128$

1.5 $2 \cdot 2^x + 1 = 1{,}25$ **1.6** $3 \cdot 3^x - 2 = 25$

2. **2.1** $3^{2x-1} = 27$ **2.2** $4^{x-1} = 4^{1-x}$

2.3 $2^{x+5} = 64$ **2.4** $5^{2x-1} = 125$

2.5 $\left(\frac{1}{4}\right)^{-x} = (4 \cdot 2^x)^{x-1}$ **2.6** $2 \cdot 3^x = 81 - 3^x$

2.7 $12^{2x} = 8(36^x)$ **2.8** $4^x + 16 = 3(2^{2x})$

3. **3.1** $\frac{4^{2x-1}}{8^x} = 2^{2-x}$ **3.2** $\frac{5^{x-1}25^x}{5^{x+2}} = 1$

3.3 $\frac{9^x - 1}{3^x + 1} = 8$ **3.4** $\frac{4^x - 1}{2^x + 1} = 15$