Exponents

Laws of exponents; Calculations

Reminder

Law 1: Multiplication of powers with the same base: For $a \neq 0$ we can write the following:
$a^m \times a^n = a^{m+n}$, but $a^m \times b^n \neq (ab)^{m+n}$ (because the bases differ).

Law 2: Division of powers with the same base: For $a \neq 0$ we can write the following:

If $m > n$ then $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

If $m = n$ then $a^m \div a^m = \frac{a^m}{a^m} = 1$ or $a^m \div a^m = a^{m-m} = a^0$, which means that $a^0 = 1$

If $m < n$ then $a^m \div a^n = \frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

If $m = 0$ then $a^0 \div a^n = \frac{a^0}{a^n} = \frac{1}{a^n}$ or $a^0 \div a^n = a^{0-n} = a^{-n}$, which means that $a^{-n} = \frac{1}{a^n}$

Law 3: Raising a power to a power: For $a \neq 0$ we can write $(a^m)^n = a^{m \times n}$.

Law 4: Finding a power of a product in brackets: For $a \neq 0$ and $b \neq 0$ we can write $(ab)^n = a^n \times b^n$.

1. Simplify the following expressions. Write all answers with positive exponents.

a) $x^4 \times x^3$ **b)** $x^4 \div x^3$ **c)** x^0

d) $y^2 \times y^5$ **e)** $y^2 \div y^5$ **f)** y^{-3}

g) $a^{-7} \times a^3$ **h)** $\frac{b^3}{b^{-3}}$ **i)** $(2x)^3$

j) $\frac{1}{a^{-7} \times a^3}$ **k)** $\frac{3^0 a^4 b^5}{a^5 b^6}$ **l)** $((x^2)^3)^2$

m) $\frac{(3x)^5}{(3x)^3}$ **n)** $(3^0 x^2 y)^4$ **o)** $\frac{26m^2n^3}{13mn}$

2. Simplify the following expressions. Write all answers with positive exponents.

a) $x^5y^9x^6y^{-8}$

b) $\left[(xy^2z^3)^2\right]^3$

c) $\frac{x^5y^9}{x^6y^8}$

d) $\frac{-24x^5}{16x^4}$

e) $(2s)^4.(-7s^2)^2$

f) $\frac{y^2y^{-3}y^4}{y^{-2}}$

g) $\frac{-5a^3}{2a.5a^4}$

h) $a^4b^3c^2 \times a^{-2}b^4c^{-6}$

i) $(ab)^2(a^2b)$

j) $\frac{y^4z^{-9}}{x^5y^{-7}}$

k) $\frac{6x^3y^4 \times -2x^2y}{xy^3 \times 4x^3}$

Exponents

Solving problems

Examples

1. Solve $3^x = 9$.
   $3^x = 9$
   $= 3^2$ (because $9 = 3 \times 3$)
   $\therefore x = 2$ (bases are equal)

2. Solve $3^x = \frac{1}{9}$.
   $3^x = \frac{1}{9}$
   $= \frac{1}{3^2}$ (because $9 = 3 \times 3$)
   $= 3^{-2}$ (law of exponents)
   $\therefore x = -2$ (bases are equal)

3. Solve $3^{x-1} = \frac{1}{27}$.
   $3^{x-1} = \frac{1}{27}$
   $= \frac{1}{3^3}$ (because $27 = 3 \times 3 \times 3$)
   $= 3^{-3}$ (law of exponents)
   $\therefore x - 1 = -3$ (bases are equal)
   $\therefore x = -3 + 1 = -2$

1. Solve for x.

a) $2^x = 32$

b) $3^x = 1$

c) $4^x = 256$

d) $5^x = \frac{1}{625}$

e) $6^x = \frac{1}{216}$

f) $7^x = \frac{1}{343}$

g) $8^{x+1} = 64$

h) $9^{x-1} = \frac{1}{729}$

i) $3^{2x-6} = 1$

2. A man had two children. In turn they each had two children. This pattern was repeated twelve times from generation to generation.

a) How many children were there in the sixth generation?

b) Which generation will consist of 1 024 children?
[Hint: Let the number of the generation be x.]

Tip

You are the first generation of your parents and the second generation of your grandparents.

3. Express the following numbers in scientific notation.

a) In 2020 the world population will be about 7 585 million.

b) The diameter of an oxygen atom is 0,000 000 000 12 m.