

Algebraic equations

Solving equations using inspection and inverse operations

Exemplar

1. Solve $3(x - 2) = 12$ using inspection.

Ask: 3 times which number equals 12?

The answer is 4, so $x - 2 = 4$.

Ask: Which number minus 2 equals 4?

The answer is 6, so $x = 6$.

Check: LHS = $3(x - 2) = 3(6 - 2) = 3(4)$
 $= 12 = \text{RHS}$

3. Solve $(x - 1)(x + 3) = 0$ using inspection.

RHS = 0 if at least one factor of LHS equals 0

$\therefore x - 1 = 0$ or $x + 3 = 0$

$\therefore x = 1$ or $x = -3$

2. Solve $3(x - 2) = x + 2$ using inverse operations.

$$3(x - 2) = x + 2$$

$$\therefore 3x - 6 = x + 2 \quad (\text{expand LHS})$$

$$\therefore 3x - x - 6 = x - x + 2 \quad (\text{add } -x \text{ to LHS and RHS})$$

$$\therefore 2x - 6 = 2$$

$$\therefore 2x - 6 + 6 = 2 + 6 \quad (\text{add 6 to LHS and RHS})$$

$$\therefore 2x = 8$$

$$\therefore \frac{2x}{2} = \frac{8}{2} \quad (\text{divide LHS and RHS by 2})$$

$$\therefore x = 4$$

Check: LHS = $3(x - 2) = 3(4 - 2) = 3(2) = 6$

RHS = $x + 2 = 4 + 2 = 6$; therefore LHS = RHS

1. Solve the following algebraic equations using inspection.

a) $x + 6 = 9$

b) $x - 8 = 12$

c) $15 - x = 10$

d) $-5 + x = 4$

e) $2x = 14$

f) $-7x = 28$

g) $5(x + 4) = 30$

h) $5(x - 8) = 30$

i) $(x + 5)(x - 7) = 0$

j) $x(x + 0,5) = 0$

k) $(2x - 4)(3x + 9) = 0$

l) $xyz = 0$

2. Solve the following algebraic equations using inverse operations.

a) $x + 6 = -9$

b) $x - 8 = -12$

c) $15 - x = -10$

d) $-5 + x = -4$

e) $-2x = 14$

f) $-7x = -28$

g) $5a + 4 = 3(a + 3) + 1$

h) $8x - 13 = 5x - (6x - 5)$

i) $3(m - 2) = 5(m + 2)$

j) $3k = 9k - (2k - 8)$

k) $-4(2x - 5) = -4x + 10$

l) $2(p - 3) = p - 3(p + 2)$

Algebraic equations

Solving equations with fractions; Solving problems

Examples

- Solve $\frac{x+1}{3} = 4$ using inspection.
Ask: Which number divided by 3 equals 4?
The answer is 12, so $x + 1 = 12$.
Ask: Which number plus 1 equals 12?
The answer is 11, so $x = 11$.
Check: LHS = $\frac{x+1}{3} = \frac{11+1}{3} = \frac{12}{3} = 4$
= RHS
- Solve $\frac{a}{2} + \frac{2a}{3} - \frac{3a}{4} = 5$ using inverse operations.
 $\frac{a}{2} + \frac{2a}{3} - \frac{3a}{4} = \frac{5}{1}$ (write each term as a fraction)
The LCM of all the denominators is 12.
 $\frac{6a}{12} + \frac{8a}{12} - \frac{9a}{12} = \frac{60}{12}$ (write each denominator as 12)
 $\frac{6a + 8a - 9a}{12} = \frac{60}{12}$ (write single fraction on both sides)
 $\therefore 6a + 8a - 9a = 60$ (numerators must be equal)
 $\therefore 5a = 60$ (simplify LHS)
 $\therefore a = 12$ (divide both sides by 5)
Check: LHS = $\frac{12}{2} + \frac{24}{3} - \frac{36}{4} = 6 + 8 - 9 = 5 = \text{RHS}$

1. Solve the following algebraic equations using inspection.

- | | | |
|---------------------------------------------|---------------------------------------------|----------------------------------------------|
| a) $\frac{x}{3} = 12$ <input type="text"/> | b) $\frac{x}{-4} = 8$ <input type="text"/> | c) $\frac{x}{5} = -4$ <input type="text"/> |
| d) $\frac{x}{-6} = -7$ <input type="text"/> | e) $\frac{2x}{3} = 4$ <input type="text"/> | f) $\frac{3x}{-4} = 6$ <input type="text"/> |
| g) $\frac{x+3}{2} = 6$ <input type="text"/> | h) $\frac{x-3}{4} = 9$ <input type="text"/> | i) $\frac{9+2x}{3} = 4$ <input type="text"/> |

2. Solve the following algebraic equations using inverse operations.

- | | | |
|------------------------------------------------------------------|-----------------------------------------------------------------------------|--------------------------------------------------------------------------|
| a) $\frac{x}{3} = -6$
<input type="text"/> | b) $\frac{x}{-4} = 7$
<input type="text"/> | c) $\frac{x}{5} = -9$
<input type="text"/> |
| d) $\frac{x}{-6} = -11$
<input type="text"/> | e) $\frac{-2x}{3} = 8$
<input type="text"/> | f) $\frac{3x}{-4} = 9$
<input type="text"/> |
| g) $\frac{2}{5}n - 2 = \frac{1}{5}n + 4$
<input type="text"/> | h) $\frac{7b}{10} + \frac{b}{2} - \frac{4b}{5} = 4$
<input type="text"/> | i) $\frac{1}{3}(9x + 15) = \frac{2}{5}(20x - 5)$
<input type="text"/> |

3. Write an equation for each problem and then solve it.

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|-----------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| a) The sum of three consecutive whole numbers is 111. Find the numbers.

<input type="text"/> | b) A father is 4 times older than his son. In 16 years he will be twice as old as his son. How old is the son now?

<input type="text"/> |
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